The leading and most up-to-date textbook on the far-ranging algorithmic methodology of Dynamic Programming, which can be used for optimal control, Markovian decision problems, planning and sequential decision making under uncertainty, and discrete/combinatorial optimization. The treatment focuses on basic unifying themes, and conceptual foundations. It illustrates the versatility, power, and generality of the method with many examples and applications from engineering, operations research, and other fields. It also addresses extensively the practical application of the methodology, possibly through the use of approximations, and provides an extensive treatment of the far-reaching methodology of Neuro-Dynamic Programming/Reinforcement Learning.

The first volume is oriented towards modeling, conceptualization, and finite-horizon problems, but also includes a substantive introduction to infinite horizon problems that is suitable for classroom use. The second volume is oriented towards mathematical analysis and computation, treats infinite horizon problems extensively, and provides an up-to-date account of approximate large-scale dynamic programming and reinforcement learning. The text contains many illustrations, worked-out examples, and exercises.

This extensive work, aside from its focus on the mainstream dynamic programming and optimal control topics, relates to our Abstract Dynamic Programming (Athena Scientific, 2013), a synthesis of classical research on the foundations of dynamic programming with modern approximate dynamic programming theory, and the new class of semicontractive models, Stochastic Optimal Control: The Discrete-Time Case (Athena Scientific, 1996), which deals with the mathematical foundations of the subject Neuro-Dynamic Programming (Athena Scientific, 1996), which develops the fundamental theory for approximation methods in dynamic programming, and Introduction to Probability (2nd Edition, Athena Scientific, 2008), which provides the prerequisite probabilistic background.

New features of the 4th edition of Vol. I (see the Preface for details):

- provides textbook accounts of recent original research on approximate DP, limited lookahead policies, rollout algorithms, model predictive control, Monte-Carlo tree search and the recent uses of deep neural networks in computer game programs such as Go.
- includes a substantial number of new exercises, detailed solutions of many of which are posted on the internet (see below).

New features of the 4th edition of Vol. II (see the Preface for details):

- Contains a substantial amount of new material, as well as a reorganization of old material. The length has increased by more than 60% from the third edition, and most of the old material has been restructured and/or revised. Volume II now numbers more than 700 pages and is larger in size than Vol. I. It can arguably be viewed as a new book!
- A major expansion of the discussion of approximate DP (neuro-dynamic programming), which allows the practical application of dynamic programming to large and complex problems. Approximate DP has become the central focal point of this volume.
- Extensive new material, the outgrowth of research conducted in the six years since the previous edition, has been included.
- The first account of the emerging methodology of Monte Carlo linear algebra, which extends the approximate DP methodology to broadly applicable problems involving large-scale regression and systems of linear equations.
- Expansion of the theory and use of contraction mappings in infinite state space problems and in neuro-dynamic programming.

Review of Vol. I, 4th Edition:

"Prof. Bertsekas book is an essential contribution that provides practitioners with a 30,000 feet view in Volume I - the second volume takes a closer look at the specific algorithms, strategies and heuristics used - of the vast literature generated by the diverse communities
that pursue the advancement of understanding and solving control problems. This is achieved through the presentation of formal models for special cases of the optimal control problem, along with an outstanding synthesis (or survey, perhaps) that offers a comprehensive and detailed account of major ideas that make up the state of the art in approximate methods. The book ends with a discussion of continuous time models, and is indeed the most challenging for the reader. Still I think most readers will find there too at the very least one or two things to take back home with them.

Each Chapter is peppered with several example problems, which illustrate the computational challenges and also correspond either to benchmarks extensively used in the literature or pose major unanswered research questions. At the end of each Chapter a brief, but substantial, literature review is presented for each of the topics covered.

This is a book that both packs quite a punch and offers plenty of bang for your buck. Graduate students wanting to be challenged and to deepen their understanding will find this book useful. PhD students and post-doctoral researchers will find Prof. Bertsekas' book to be a very useful reference to which they will come back time and again to find an obscure reference to related work, use one of the examples in their own papers, and draw inspiration from the deep connections exposed between major techniques. Undergraduate students should definitely first try the online lectures and decide if they are ready for the ride."

Miguel, at Amazon.com, 2018.

Review of Vol. II, 4th Edition:
" This is an excellent textbook on dynamic programming written by a master expositor. Between this and the first volume, there is an amazing diversity of ideas presented in a unified and accessible manner. This new edition offers an expanded treatment of approximate dynamic programming, synthesizing a substantial and growing research literature on the topic. "


Among its special features, the book:
- provides a unifying framework for sequential decision making
- treats simultaneously deterministic and stochastic control problems popular in modern control theory and Markovian decision processes popular in operations research
- develops the theory of deterministic optimal control problems including the Pontryagin Minimum Principle
- introduces recent suboptimal control and simulation-based approximation techniques (neuro-dynamic programming), which allow the practical application of dynamic programming to complex problems that involve the dual curse of large dimension and lack of an accurate mathematical model
- provides a comprehensive treatment of infinite horizon problems

Reviews of Pre-2005 Editions:

Review of Vols. I and II, 3rd Edition:
"In conclusion, the new edition represents a major upgrade of this well-established book. The coverage is significantly expanded, refined, and brought up-to-date. This is the only book presenting many of the research developments of the last 10 years in approximate DP/neuro-dynamic programming/reinforcement learning (the monographs by Bertsekas and Tsitsiklis, and by Sutton and Barto, were published in 1996 and 1998, respectively). The book is a rigorous yet highly readable and comprehensive source on all aspects relevant to DP: applications, algorithms, mathematical aspects, approximations, as well as recent research. It should be viewed as the principal DP textbook and reference work at present. With its rich mixture of theory and applications, its many examples and exercises, its unified treatment of the subject, and its polished presentation style, it is eminently suited for classroom use or self-study."


Review of Vol. I, 3rd Edition:
"In addition to being very well written and organized, the material has several special features that make the book unique in the class of introductory textbooks on dynamic programming. For instance, it presents both deterministic and stochastic control problems, in both discrete- and continuous-time, and it also presents the Pontryagin minimum principle for deterministic systems together with several extensions. It contains problems with perfect and imperfect information, as well as minimax control methods (also known as worst-case control problems or games against nature). I also has a full chapter on suboptimal control and many related techniques, such as open-loop feedback controls, limited lookahead policies, rollout algorithms, and model predictive control, to name a few. ... In conclusion the book is highly recommendable for an introductory course on dynamic programming and its applications."

Onesimo Hernandez Lerma, in Mathematical Reviews, Issue 2006g.

"In conclusion, this book is an excellent source of reference ... The main strengths of the book are the clarity of the exposition, the quality and variety of the examples, and its coverage of the most recent advances."

Thomas W. Archibald, in IMA Jnl. of Mathematics Applied in Business & Industry

"Here is a tour-de-force in the field."
"By its comprehensive coverage, very good material organization, readability of the exposition, included theoretical results, and its challenging examples and exercises, the reviewed book is highly recommended for a graduate course in dynamic programming or for self-study. It is a valuable reference for control theorists, mathematicians, and all those who use systems and control theory in their work. Students will for sure find the approach very readable, clear, and concise. Misprints are extremely few."

Vasile Sima, in SIAM Review

"In this two-volume work Bertsekas caters equally effectively to theoreticians who care for proof of such concepts as the existence and the nature of optimal policies and to practitioners interested in the modeling and the quantitative and numerical solution aspects of stochastic dynamic programming."

Michael Caramanis, in Interfaces

"The textbook by Bertsekas is excellent, both as a reference for the course and for general knowledge. It is well written, clear and helpful"

Student evaluation guide for the Dynamic Programming and Stochastic Control course at the Massachusetts Institute of Technology

The author is McAfee Professor of Engineering at the Massachusetts Institute of Technology and a member of the prestigious US National Academy of Engineering. He is the recipient of the 2001 A. R. Raggazini ACC education award, the 2009 INFORMS expository writing award, the 2014 Kachiyan Prize, the 2014 AACC Bellman Heritage Award, and the 2015 SIAM/MOS George B. Dantzig Prize. He has been teaching the material included in this book in introductory graduate courses for more than forty years.

**Supplementary Material:**

The material listed below can be freely downloaded, reproduced, and distributed.

- Videos on Approximate Dynamic Programming. A 6-lecture, 12-hour short course, Tsinghua University, Beijing, China, 2014
- Lecture slides for a 6-lecture short course on Approximate Dynamic Programming, Tsinghua University, Beijing, China, 2014
- Approximate Finite-Horizon DP videos and slides(4-hours) A 4-lecture series from the author's web site, 2017
- Prof. Bertsekas' Course Lecture Slides, 2004
- Prof. Bertsekas' Course Lecture Slides, 2015
- Theoretical problem solutions, Volume 1
- Course Material at Open Courseware at MIT
- Material from 3rd edition of Vol. I that was not included in the 4th edition Minimum Variance Control (Section 5.3 and Appendix F) and Adaptive Control (Section 6.1)
- Prof. Bertsekas' Research Papers on Dynamic and Neuro-Dynamic Programming
- Prof. Bertsekas' Ph.D. Thesis at MIT, 1971, Control of Uncertain Systems with a Set-Membership Description of the Uncertainty, which contains supplementary material for Vol. 1
- Errata

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